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Schrödinger's Cat Meets Einstein's Twins: A Superposition of Different Clock Times

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Abstract The phenomenon of quantum superposition, which allows a physical system to exist in different states 'simultaneously', is one of the most bizarre notions in physics. Here we illustrate an even more bizarre example of it: a superposed state of a physical system consisting of both an 'older' version and a 'younger' version of that system. This can be accomplished by exploiting the special relativistic effect of time dilation featuring in Einstein's famous twin paradox.

Keywords Superposition · Schroedinger's cat · Twin paradox

The superposition principle is the cornerstone of quantum physics. It stipulates that if a physical system can exist in two different states, then it can also exist in a superposition of these two states. For example, an atom can be located at two different positions within a solid, but the superposition principle implies that the atom can also exist in both of these locations 'simultaneously'. The existence of superpositions has been tested and confirmed experimentally not only with atoms, but also with other systems, such as photons [1]. The possibility of superpositions of large scale systems has been hotly debated, most notably by Schrödinger who tried to expose its paradoxical nature by suggesting a superposition of a dead and alive cat [2]. The domain of validity of the superposition principle, however, is still an open question in physics, and has great significance for an attempt to solve the measurement problem [3].

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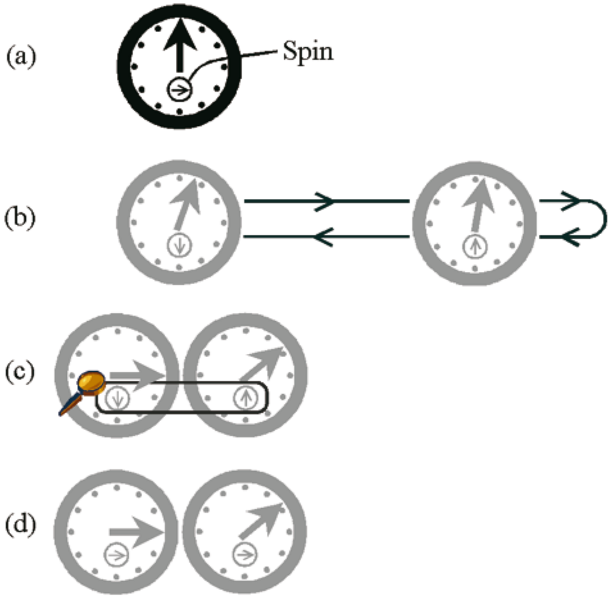

Fig. 1 The schematic representation of the superposition of the different clock times. The clock in the figure represents a physical system together with an additional internal two-dimensional degree of freedom which we refer to as spin. (**a**) Firstly, we prepare the spin in a superposed state of its basis states ‘up’ and ‘down’. This superposition is represented with an arrow pointing to the right. (**b**) Secondly, we boost the clock conditionally on its internal state being ‘up’, and this branch of the clock then makes a round trip returning to its original position. (**c**) At the end of the trip the system is in an entangled state where the spin is quantumly correlated to the clock time: $|\text{up}\rangle|\text{younger}\rangle + |\text{down}\rangle|\text{older}\rangle$. Now we make a measurement on the spin, represented in the figure by a magnifying glass, in the basis $|\text{up}\rangle \pm |\text{down}\rangle$, whose purpose is to disentangle the spin variable from the clock time variable

Here we discuss yet another simple but bizarre example of quantum superposition, that of two different states of a cat, each of which has a different age in spite of being located at the same point in space. Thus, we illustrate another way of appreciating the weirdness of quantum superposition. We imagine that two different states in a quantum superposition are treated like the twins in Einstein’s famous ‘twin paradox’ in special relativity [4]. Namely, one of the states in the superposition is taken on a journey ‘into space and back’ while the other one remains stationary (see Fig. 1). Because of the time dilation effect of relativity, the state which is taken on a trip experiences a slowing down of time compared to the stationary state and will actually be younger at the end of the trip. Therefore, assuming that both quantum physics and relativity are correct (and no experiment so far indicates otherwise), the final state of the system will be in a superposition of the younger version and the older one of the same system. Time here is measured purely operationally using a local clock pertaining to the system just as in special relativity (see Fig. 1).

Here is how this superposition can be accomplished in more detail. We need a physical system with an additional internal two-dimensional degree of freedom like the spin of an electron. First, we need to prepare a superposition of the spin states, which we will call ‘up’ and ‘down’. This is a standard experiment and can easily be accomplished with almost any simple quantum system. The next step is to apply a boost to the system conditional on its internal state. If the internal state of the system is ‘up’ then the whole system undergoes the boost; otherwise, if the spin is ‘down’, the system undergoes no transformation apart from the usual temporal free evolution and it always stays in the same spatial position. Then, the boosted system is returned back to the original position. According to the theory of special relativity, the 'boosted branch' of the system is at the end of the trip younger than the other (stationary) branch. Therefore the system is now in the following entangled state (we omit the normalization factor): $|\text{up}\rangle|\text{younger}\rangle + |\text{down}\rangle|\text{older}\rangle$ (apart from some small corrections due to quantum relativistic effects that will be discussed below). If we finally make a measurement of the spin in the $|\text{up}\rangle \pm |\text{down}\rangle$ basis, the system is left in the state $|\text{younger}\rangle \pm |\text{older}\rangle$. This state is, amazingly, an equal superposition of two different age states of one and the same physical system. This argument can be naturally extended to superpositions of arbitrary many clock times by using a higher-dimensional internal degree of freedom.

How can we test the fact that the system is in this kind of 'temporal' superposition rather than in a statistical mixture of the two different states? This can be performed by analogy with ordinary (spatial) interferometers. We need another round of the same experiment with the roles of the spin states being interchanged before the final measurement on the internal degree of freedom in the $|\text{up}\rangle \pm |\text{down}\rangle$ basis. Now the spin 'down' is boosted while the spin 'up' remains stationary. At the end of the protocol, if the state is in the superposition, both branches of the system will have the same age which is just the sum of the younger age and the older one, and the entire state becomes $(|\text{up}\rangle + |\text{down}\rangle)|\text{younger} + \text{older}\rangle$. If, on the other hand, the state is in a statistical mixture, then the final state becomes $(|\text{up}\rangle\langle\text{up}| + |\text{down}\rangle\langle\text{down}|)|\text{younger} + \text{older}\rangle\langle\text{younger} + \text{older}|$, in which case the internal spin is in a statistical mixture. Therefore, by repeating this experiment a number of times and observing the internal spin, we can test whether the system is actually in the superposition.

The gedankenexperiment just mentioned is quite similar to that proposed in [5]. However, our scenario is different from it in that the spin degree of freedom conditioning the boost is included in the boosted system itself and we disentangle it with the system at the end by making the final measurement in the $|\text{up}\rangle \pm |\text{down}\rangle$ basis. In our gedankenexperiment, the 'space-time' inside of the boosted system is completely in a superposition of different 'eras' as long as it is isolated from the outside world. In other words, one branch has a longer history than the other.

We note that in the above analysis we have made one further assumption: the relativistic effects involved in achieving the boost, such as the necessary acceleration and deceleration of the system, are minimal and can be ignored. Even under perfect conditions, the accelerated branch of the system will still be experiencing noise from the environment due to the Unruh effect [6]: any accelerated system in the vacuum experiences a local thermal bath whose temperature is proportional to the acceleration. However, for any reasonable accelerations this noise is small and certainly not enough to completely destroy the 'temporal' superposition described above. Even for most extreme accelerations we still have at least one half probability of observing this temporal superposition.

It may be tempting to view the preparation of different clock times in the following way. We take a physical system and fully estimate its state at one time using standard methods of quantum tomography. Then we let the system evolve for some time and at the end of its evolution we fully estimate its state again. With this knowledge we can now, at least in principle, prepare the superposition of the two estimated states. This would correspond to the superposition of two different ages just like in our original argument. However, the full state tomography requires multiple copies of the same system, and it is quantumly impossible to infer the state of the system from just one copy. This makes our twin based superposition of clock times even more striking because we have shown how to prepare it without any knowledge of the state of the system at any stage of the experiment apart from the control of the (internal) spin.

Finally, let us describe a natural but non-trivial consequence of the possibility of superposition of different clock times. Suppose we perform the same experiment with two quantum systems instead of just one. Then, the final state of the entire system becomes an entangled state such as $|\text{younger}\rangle|\text{younger}\rangle + |\text{older}\rangle|\text{older}\rangle$, which clearly implies that the 'temporal' degrees of freedom like ages can be entangled in quantum physics. This suggests that the notion of entanglement can be exhibited in far more bizarre ways than in usual arguments, but far-reaching implications of this consequence are well beyond the scope of this paper.

We believe that this exposition not only gives another simple and pedagogical way of appreciating the weirdness of the superposition principle, but also offers a viewpoint on issues regarding the quantum mechanical meaning of time [7]. The astonishing success of quantum physics in the last hundred years has, somewhat surprisingly, had very little effect on our understanding of this notion. However, if—in the spirit of relativity—we treat space and time on an equal footing, then the concept of superposition in time is quantum mechanically inevitable. Our illustration partly supports the view that different instances of time could be viewed as different superposition elements in a certain macroscopic wave function [8]. We hope that our gedankenexperiment stimulates further investigation into the quantum meaning of space and time.

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